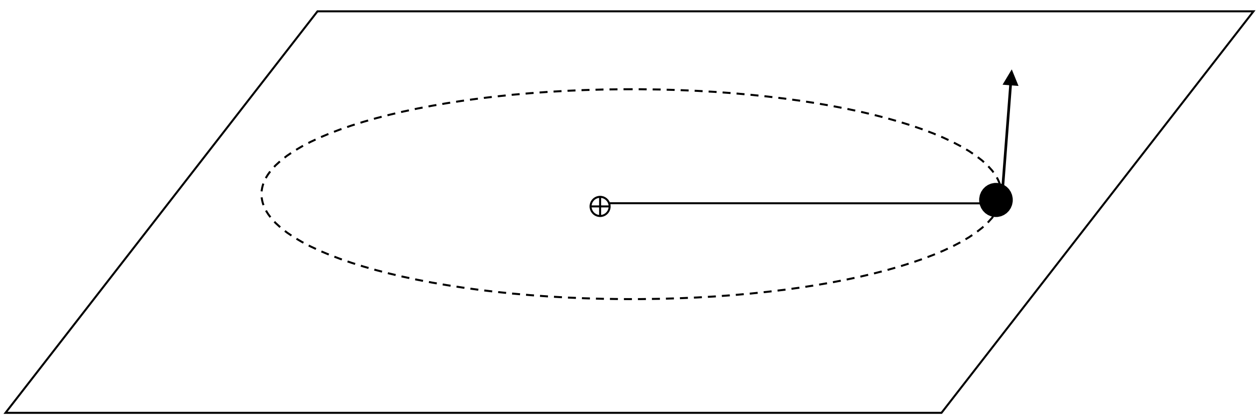
Problem for week 43
Circular motion

(a) Describe the conditions for circular motion to take place.

(b) A mass of 2.0 kg is attached to a string. The string is horizontal. The mass is released.

Determine the tension in the string when the ball is at its lowest point in its path.

(c) The ball in (b) is tied to a string and moves along a circle of radius $R = 0.44$ m on a frictionless **horizontal** table with constant speed $v = 11$ m s^{-1}. The string breaks when the tension in it exceeds 750 N.



(i) State why the ball is **not** in equilibrium.

(ii) Calculate the number of revolutions per second made by the ball.

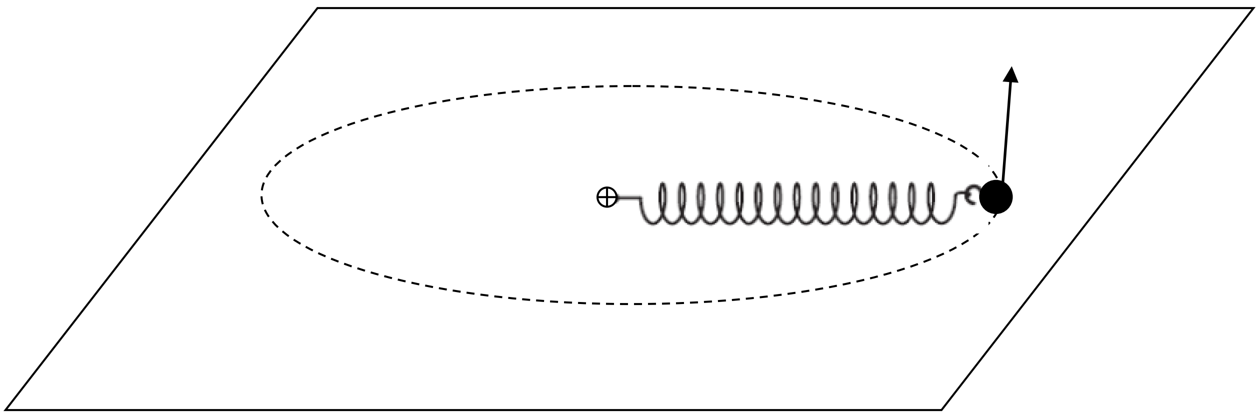
(iii) Calculate the angular speed of the ball.

(iv) Determine the tension in the string.

(v) Calculate the maximum speed at which the ball can move without breaking the string.

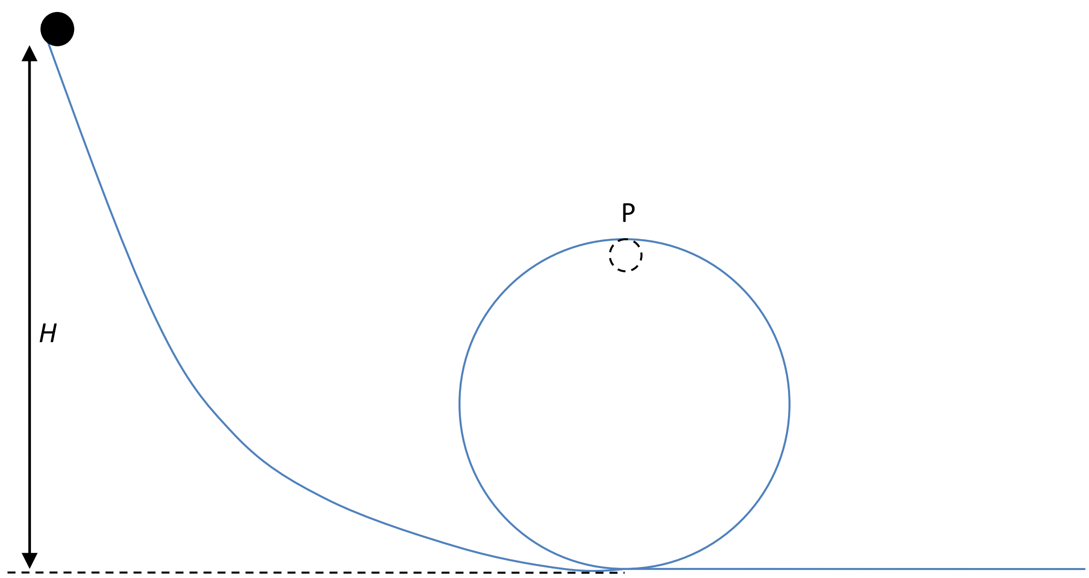
(vi) The ball has a speed of 11 m s^{-1}. Determine the minimum length of the string so that the string does not break.

(d) The string in (c) is replaced by a spring. As the ball rotates the spring is extended by an amount equal to $\frac{L_0}{5}$ where L_0 is the natural length of the spring.



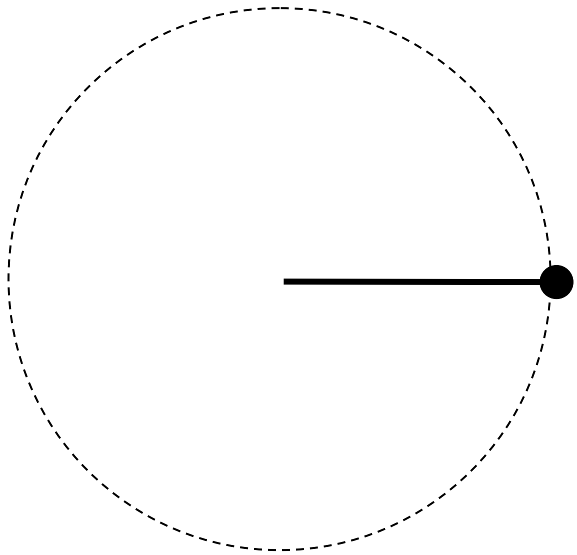
Determine the ratio kinetic energy of ball to elastic potential energy of the spring.

(e) The diagram shows a loop-the-loop toy. The radius of the circle is 0.20 m.



The ball is released from rest at a height *H* from the ground. Determine the minimum value of *H* so that the ball never loses contact with the track at point P.

(f) The ball in (b) is now tied to the end of a rigid rod of length 0.45 m and rotates on a **vertical** circle with **constant** speed 4.0 m s^{-1}. At the position shown the rod is horizontal.



(i) Draw the forces on the ball at the position shown.

(ii) Calculate the tension in the rod at the position shown.

Answers

(a) The mass must have a non-zero velocity and a force of constant magnitude always at right angles to the velocity.

(b) At the lowest point the resultant force is $T-mg$ and so $T-mg=\frac{mv^2}{R}$ where R is the length of the string. By energy conservation: $mgR=\frac{1}{2}mv^2 \Rightarrow mv^2=2mgR$. Hence,

$$T=mg+\frac{mv^2}{R}=mg+\frac{2mgR}{R}=3mg=3\times 2.0\times 9.8=58.8\approx 59\text{ N}.$$

(c)

(i) There is a non-zero net force on the ball/the ball does not move with constant velocity because the direction is changing.

(ii) One revolution is completed in $\frac{2\pi R}{v}=\frac{2\pi\times 0.44}{11}=0.2513\text{ s}$ so in 1 second the number of revolutions is $\frac{1}{0.2513}=3.98\approx 4.0$.

(iii) $\omega=\frac{2\pi}{T}=\frac{2\pi}{0.2513}=25.00\approx 25\text{ rad s}^{-1}$.

(iv) $T=\frac{mv^2}{R}=\frac{2.0\times 11^2}{0.44}=550\text{ N}$.

(v) $\frac{mv^2}{R}=750\Rightarrow v=\sqrt{\frac{750\times 0.44}{2.0}}=12.8\approx 13\text{ m s}^{-1}$.

(vi) $\frac{mv^2}{R}=750\Rightarrow R=\frac{mv^2}{750}=\frac{2.0\times 11^2}{750}=0.323\approx 0.32\text{ m}$.

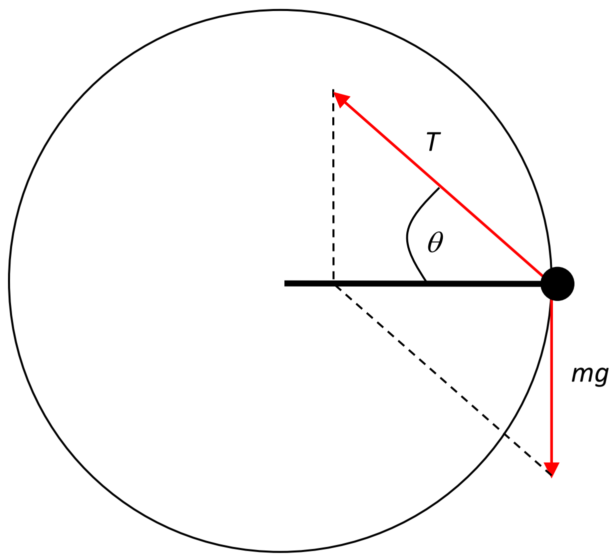
(d) From $\frac{mv^2}{r}=kx$ we have: $\frac{mv^2}{(L_0+\frac{L_0}{5})}=k\frac{L_0}{5}$, i.e. $mv^2=k\frac{L_0}{5}\frac{6L_0}{5}=\frac{6kL_0^2}{25}$. Then the required ratio is

$$\frac{K}{E_e}=\frac{\frac{1}{2}mv^2}{\frac{1}{2}k(\frac{L_0}{5})^2}=\frac{\frac{1}{2}\frac{6kL_0^2}{25}}{\frac{1}{2}k(\frac{L_0}{5})^2}=6.$$

(e) At the top point on the track the resultant force is $N+mg$ and $N+mg=\frac{mv^2}{R}$. The critical case is when $N\to 0$ in which case $v^2=gR$. By energy conservation: $mgH=mg(2R)+\frac{1}{2}mv^2$ giving $2gH=4gR+v^2\Rightarrow v^2=2gH-4gR$. Hence $gR=2gH-4gR\Rightarrow H_{min}=\frac{5R}{2}=\frac{5\times 0.20}{2}=0.50\text{ m}$.

(f)

(i) The resultant force must point towards the center of the circle so we must have:



(ii) $T\sin\theta = mg$ and $T\cos\theta = \frac{mv^2}{R}$. I.e. the resultant force is $\frac{mv^2}{R}$ along the rod. Since the tension is the hypotenuse of a right-angled triangle:

$$T = \sqrt{(mg)^2 + (\frac{mv^2}{R})^2} = \sqrt{(2.0\times 9.8)^2 + (\frac{2.0\times 4.0^2}{0.45})^2} = 73.76 \approx 74 \text{ N}.$$

(The angle is found from: $\sin\theta = \frac{mg}{T} = \frac{2.0\times 9.8}{73.76} \Rightarrow \theta = 15^\circ$.)