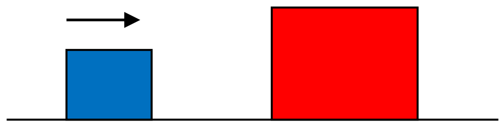
Problem for week 44

Momentum

(a) A block of mass 2.5 kg moving at 6.0 m s^{-1} collides with a stationary block of mass 7.5 kg. After the collision the two blocks move together.



(i) Calculate the common speed of the blocks after the collision.
(ii) Determine the kinetic energy that was lost in the collision.

(b) A ball of mass 0.20 kg falls vertically from rest from a height of 2.0 m. The ball rebounds to a height of 1.5 m. The ball was in contact with the floor for 0.25 s.

Determine
(i) the magnitude of the impulse delivered to the floor,
(ii) the average force that the ball exerted on the floor.

(c) A polonium nucleus (mass 210 u) decays at rest into a nucleus of lead (mass 206 u) and an alpha particle (mass 4 u).

(i) Calculate the ratio of the kinetic energy of the alpha particle to that of the lead nucleus.
(ii) The energy released in the decay is 6.5 MeV. Estimate the energy carried by the alpha particle.

(d) A block of mass M is attached to a spring on a horizontal frictionless table. The spring has its natural length. A projectile of mass m and speed u collides and gets embedded in the block.

(i) A student claims that momentum conservation cannot be applied to this collision because the spring will exert a force on the block. Explain why the student is wrong.
(ii) Find an expression for the speed of the block immediately after the collision.

The following data are available:

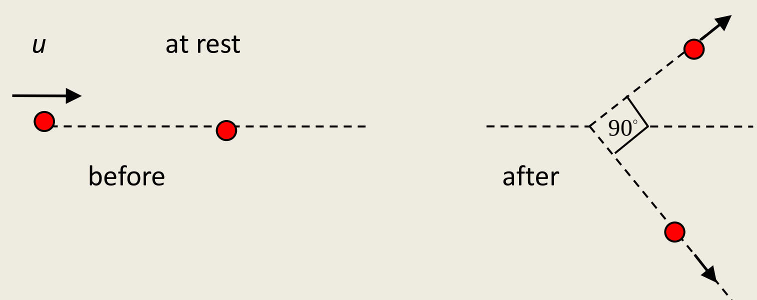
M = 4.0 kg, m = 0.050 kg, u = 220 m s^{-1}, k = 280 N m^{-1}

(iii) Determine the maximum compression of the spring.

(e) A rocket, at rest in outer space, turns on its engines ejecting gases at a rate of 50 kg s^{-1} with speed 2500 m s^{-1} relative to the rocket. The total mass of the rocket including all fuel is 6500 kg.

(i) Determine the force that will accelerate the rocket.
(ii) Estimate the initial acceleration of the rocket.
(iii) Describe how the acceleration of the rocket varies with time.

(f) A particle collides with an identical particle at rest. After the collision the angle between the velocities of the two particles is 90°.



Show that the collision is elastic.

Answers

(a)

(i) Momentum is conserved so: $2.5\times 6.0+0=(2.5+7.5)\times v \Rightarrow v=1.5\text{ m s}^{-1}$.

(ii) Kinetic energy before the collision is $\frac{1}{2}\times 2.5\times 6.0^2+0=45\text{ J}$. Kinetic energy after is $\frac{1}{2}\times 10\times 1.5^2=11.25\text{ J}$, so the lost kinetic energy is $45-11.26=33.75\approx 34\text{ J}$.

(b)

(i) Incident speed is $v_\text{i}=\sqrt{2gh}=\sqrt{2\times 9.8\times 2.0}=6.26\text{ m s}^{-1}$. Rebound speed is $v_\text{f}=\sqrt{2\times 9.8\times 1.5}=5.42\text{ m s}^{-1}$. Change in momentum is $mv_\text{f}-(-mv_\text{i})=0.20\times 5.42-0.20\times(-6.26)=2.336\approx 2.3\text{ N s}$. This is the impulse delivered to the ball and hence the same, in magnitude, delivered to the floor.

(ii) Resultant force is $N-mg=\frac{\Delta p}{\Delta t}$, so $N-0.20\times 9.8=\frac{2.336}{0.25}\Rightarrow N=11.3\approx 11\text{ N}$.

(c)

(i) The alpha particle and the lead nucleus must have equal and opposite momenta. Hence

$$\frac{K_\alpha}{K_\text{Pb}}=\frac{\frac{p^2}{2m_\alpha}}{\frac{p^2}{2m_\text{Pb}}}=\frac{m_\text{Pb}}{m_\alpha}=\frac{206}{4}=51.5.$$

(ii) $\frac{p^2}{2m_\alpha}+\frac{p^2}{2m_\text{Pb}}=6.5$, so $\frac{p^2}{2\times 4}+\frac{p^2}{2\times 206}=6.5\Rightarrow p^2\approx 51$. Hence $K_\alpha=\frac{51}{2\times 4}=6.38\approx 6.4\text{ MeV}$.

OR $K_\alpha=\frac{51.5}{52.5}\times 6.5=6.38\approx 6.4\text{ MeV}$.

(d)

(i) The student is wrong because there is no time for the spring to exert a force on the block.

(ii) $mu=(M+m)v\Rightarrow v=\frac{mu}{M+m}$.

(iii) $\frac{1}{2}kx^2=\frac{1}{2}(M+m)v^2=\frac{1}{2}(M+m)(\frac{mu}{M+m})^2=\frac{1}{2}\frac{m^2u^2}{M+m}$. Hence,

$$x=\sqrt{\frac{m^2u^2}{k(M+m)}}=\frac{mu}{\sqrt{k(M+m)}}=\frac{0.050\times 220}{\sqrt{280\times(4.0+0.050)}}=0.327\approx 0.32\text{ m}.$$

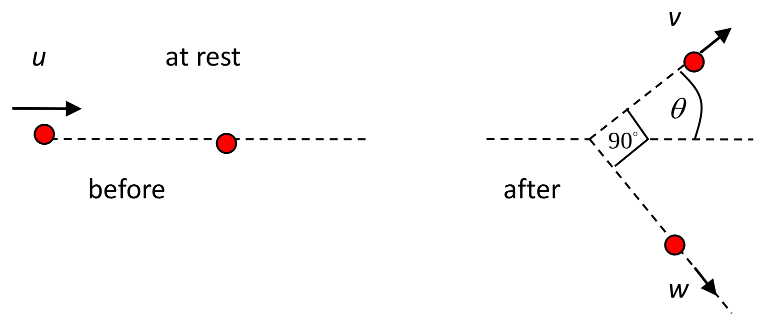
(e)

(i) The force is $F=\mu u=50\times 2500=1.25\times 10^5\text{ N}$.

(ii) The initial acceleration is $\frac{1.25\times 10^5}{6500}=19.3\approx 19\text{ m s}^{-2}$.

(iii) The acceleration increases because the force is the same but the mass decreases. The acceleration becomes zero when all the fuel is used up.

(f) One particle's velocity makes an angle with the original incident particle direction. The other makes an angle $90^\circ - \theta$.



Applying conservation of momentum along the direction of the incident particle and at right angles to it we find:

$mu = mv\cos\theta + mw\cos(90^\circ - \theta) = mv\cos\theta + mw\sin\theta$

i.e.

$u = v\cos\theta + w\sin\theta$

And

$0 = mv\sin\theta - mw\sin(90^\circ - \theta) = mv\sin\theta - mw\cos\theta$

i.e.

$0 = v\sin\theta - w\cos\theta$

We have two equations for two unknowns:

$u = v\cos\theta + w\sin\theta$

$0 = v\sin\theta - w\cos\theta$

From the second: $w = v\tan\theta$. Substituting in the first:

$u = v\cos\theta + v\tan\theta\sin\theta = (v\cos\theta + v\dfrac{\sin^2\theta}{\cos\theta}) = \dfrac{v}{\cos\theta}(\cos^2\theta + \sin^2\theta) = \dfrac{v}{\cos\theta}$. Hence, $v = u\cos\theta$.

Then, $w = v\tan\theta = u\cos\theta\tan\theta = u\sin\theta$.

The initial kinetic energy is $\dfrac{1}{2}mu^2$. The final is

$\dfrac{1}{2}mv^2 + \dfrac{1}{2}mw^2 = \dfrac{1}{2}m(u\cos\theta)^2 + \dfrac{1}{2}m(u\sin\theta)^2 = \dfrac{1}{2}mu^2(\cos^2\theta + \sin^2\theta) = \dfrac{1}{2}mu^2$.

So, the collision is elastic.

(For those we know vectors, there is a much simpler solution: let $\vec{p}$ be the initial momentum, and $\vec{p}_1$, $\vec{p}_2$ the momenta of the two particles after the collision. Then

$$\vec{p}=\vec{p}_1+\vec{p}_2$$

Thus, taking dot products:

$$\vec{p}\cdot\vec{p}=(\vec{p}_1+\vec{p}_2)\cdot(\vec{p}_1+\vec{p}_2)=\vec{p}_1\cdot\vec{p}_1+\vec{p}_2\cdot\vec{p}_2+2\vec{p}_1\cdot\vec{p}_2$$

I.e.

$p^2=p_1^2+p_2^2$ since $\vec{p}_1\cdot\vec{p}_2=0$ because the vectors are perpendicular.

Thus, $\frac{p^2}{2m}=\frac{p_1^2}{2m}+\frac{p_2^2}{2m}$ and kinetic energy is conserved.)