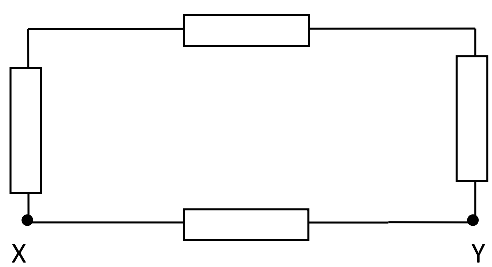
Problem of the week

Current and circuits

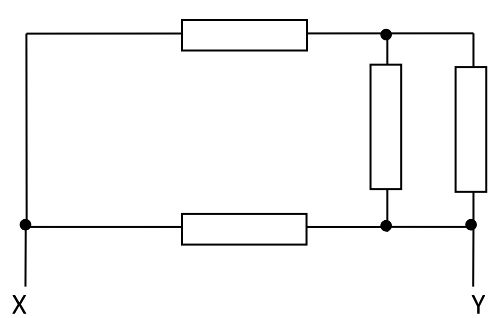
(a) X and Y are two cylindrical wires. X has resistivity ρ, radius r and length L. Y has resistivity 2ρ, radius $2r$ and length $2L$. Determine the ratio $\frac{R_X}{R_Y}$ of the resistance of X to that of Y.

(b) Each of the four resistors in circuits (i) and (ii) has resistance 20 Ω. Calculate the resistance between X and Y in each case.

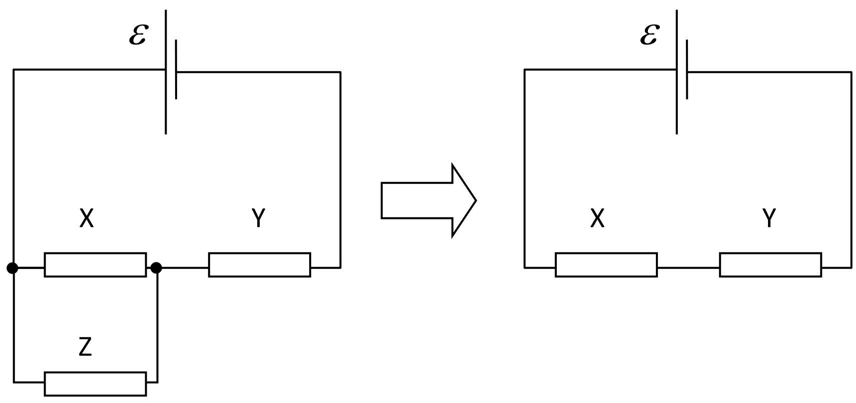
(i)



(ii)



(c) In circuit 1, X, Y and Z are identical resistors of constant resistance. When resistor Z burns out the circuit becomes circuit 2.



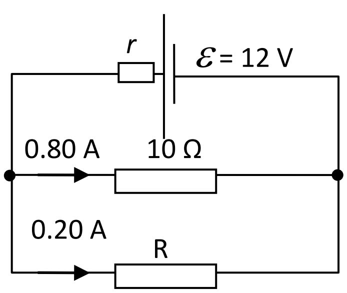
circuit 1 circuit 2

Determine the ratios

(i) $\frac{P_{X1}}{P_{X2}}$ of the power dissipated in resistor X in circuit 1 to the power in X in circuit 2,

(ii) $\frac{P_{Y1}}{P_{Y2}}$ of the power dissipated in resistor Y in circuit 1 to the power in Y in circuit 2.

(d) In the circuit shown the cell has emf $\mathcal{E}$ = 12 V and internal resistance *r*.

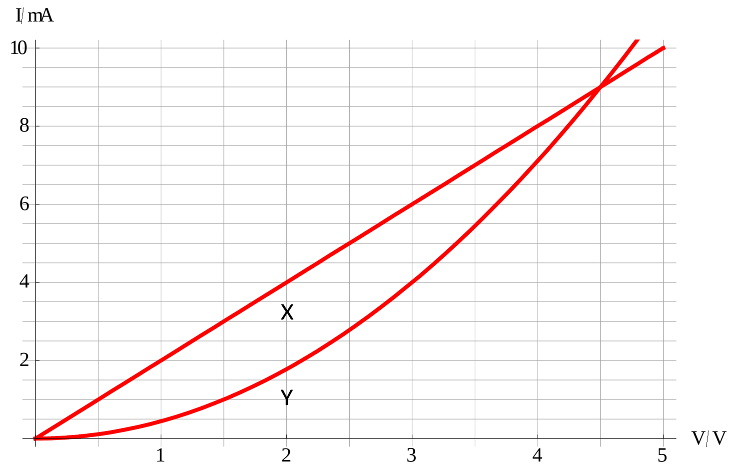


Calculate

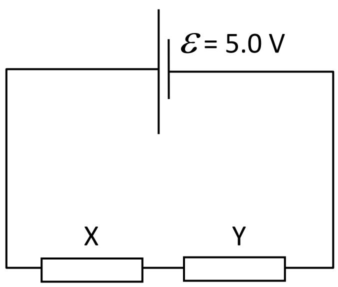
(i) *R*,

(ii) *r*.

(e) The graph shows the I-V characteristics of two resistors X and Y.



X and Y are connected in series to a cell of negligible internal resistance. The emf of the cell is 5.0 V.



(i) Explain how it may be deduced that X is an ohmic resistor.

(ii) Suggest whether the resistance of Y increases or decreases as the voltage across it increases.

(iii) Determine the ratio $\frac{P_X}{P_Y}$ of the power dissipated in resistor X to the power in resistor Y.

Answers

(a) $\frac{R_X}{R_Y} = \frac{\frac{\rho L}{A}}{\frac{2\rho \times 2L}{4A}} = \frac{\frac{\rho L}{\pi r^2}}{\frac{2\rho \times 2L}{\pi (2r)^2}} = \frac{\frac{\rho L}{\pi r^2}}{\frac{\rho L}{\pi r^2}} = 1$.

(b)

(i) The top and side resistors are in series for a total of 60 Ω. This and the lower one are in parallel for a total of $\frac{1}{60} + \frac{1}{20} = \frac{1}{15}$ i.e. 15 Ω.

(ii) The two vertical resistors are in parallel for a total of 10 Ω. This is in series with the top resistor for a total of 30 Ω. This is now in parallel with the lower one for a total of $\frac{1}{30} + \frac{1}{20} = \frac{1}{12}$ i.e. 12 Ω.

(c) In circuit 1 the total resistance is $\frac{3R}{2}$ and in circuit 2 it is 2*R*. So the current in circuit 1 in X is $\frac{1}{2} \times \frac{\varepsilon}{\frac{3R}{2}} = \frac{\varepsilon}{3R}$ and in circuit 2 it is $\frac{\varepsilon}{2R}$. Hence

(i) $\frac{P_{X1}}{P_{X2}} = \frac{R \times (\frac{\varepsilon}{3R})^2}{R \times (\frac{\varepsilon}{2R})^2} = \frac{4}{9}$. The power in X increases in circuit 2.

(ii) $\frac{P_{Y1}}{P_{Y2}} = \frac{R \times (\frac{2\varepsilon}{3R})^2}{R \times (\frac{\varepsilon}{2R})^2} = \frac{16}{9}$. The power in Y decreases in circuit 2.

(d)

(i) The voltage across R is the same as that across the 10 Ω resistor i.e. $0.80 \times 10 = 8.0\text{ V}$. Hence $R = \frac{8.0}{0.20} = 40\ \Omega$.

(ii) $V = \varepsilon - I_{\text{total}} r \Rightarrow 8.0 = 12 - 1.0r \Rightarrow r = 4.0\ \Omega$.

(e)

(i) It is ohmic because the graph is a straight line through the origin.

(ii) The current increases disproportionately more than the voltage so the resistance decreases/or evaluate resistance at 2 different voltages.

(iii) X and Y are in series, so they take the same current. The sum of the voltages across X and Y must make 5.0 V and this happens for a current of 4.0 mA giving voltages 2.0 V across X and 3.0 V across Y. Hence $\frac{P_X}{P_Y} = \frac{V_X \times I}{V_Y \times I} = \frac{2}{3}$.