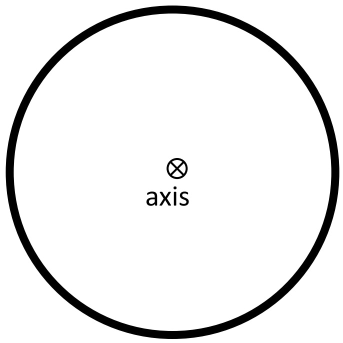
Problem of the week

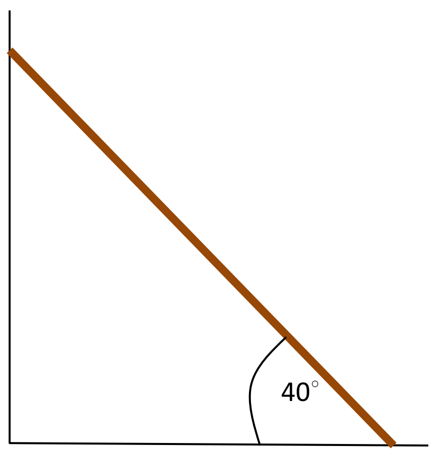
Rotational Mechanics

(a) A uniform ring has mass M and radius R.



Explain why the moment of inertia of the ring about the axis shown is MR^2.

(b) A uniform ladder of weight W = 250 N and length L rests against a smooth wall. The floor is rough. The static coefficient of friction between the ladder and the floor is 0.70. The ladder makes an angle 40° to the horizontal.



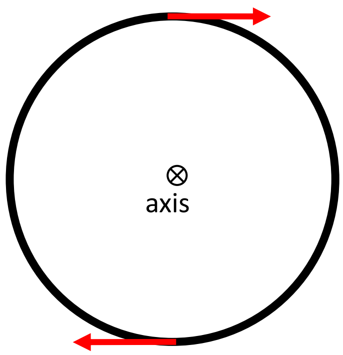
(i) Draw the forces on the ladder.

(ii) Calculate the frictional force acting on the ladder.

(iii) A string is hung from the middle of the ladder and a block of weight 50 N is attached at the other end, so it hangs vertically. Determine if the ladder will still be in equilibrium.

(iv) The string and the hanging weight are removed. Determine the smallest angle the ladder can make with the horizontal and still be in equilibrium.

(c) Two equal and opposite forces of magnitude 120 N act on the ring as shown. M = 35 kg and R = 0.80 m.

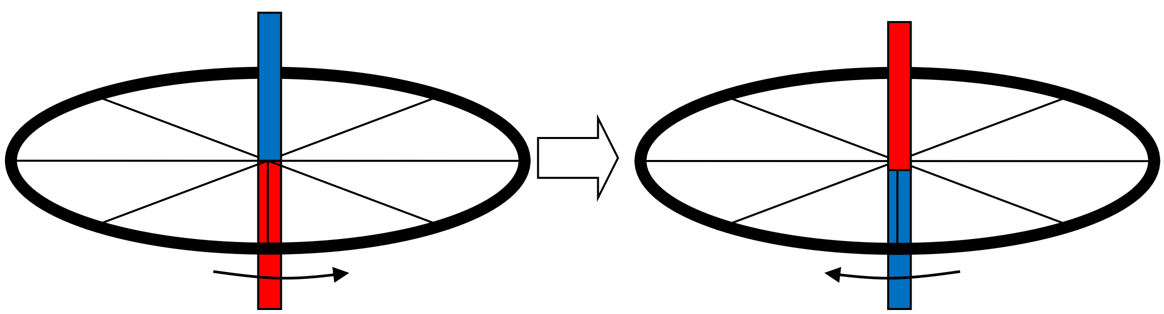


(i) Suggest whether the center of mass of the ring will move or not.

(ii) Determine the angular acceleration of the ring.

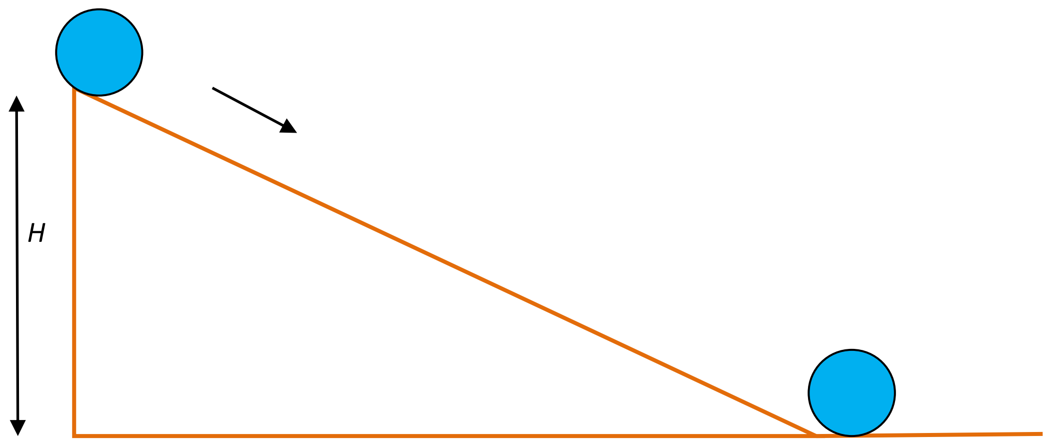
(iii) The forces act for 3.0 s. The direction of the forces is always tangential to the ring. Determine the change in the angular momentum of the ring.

(d) A wheel with its plane horizontal rotates about a vertical axis through its center. The angular momentum of the wheel about the rotation axis is 15 kg m^2 s^{-1}.



The wheel is turned upside down in 2.0 s. The magnitude of the angular momentum of the wheel stays the same. Calculate the average torque that was exerted on the wheel.

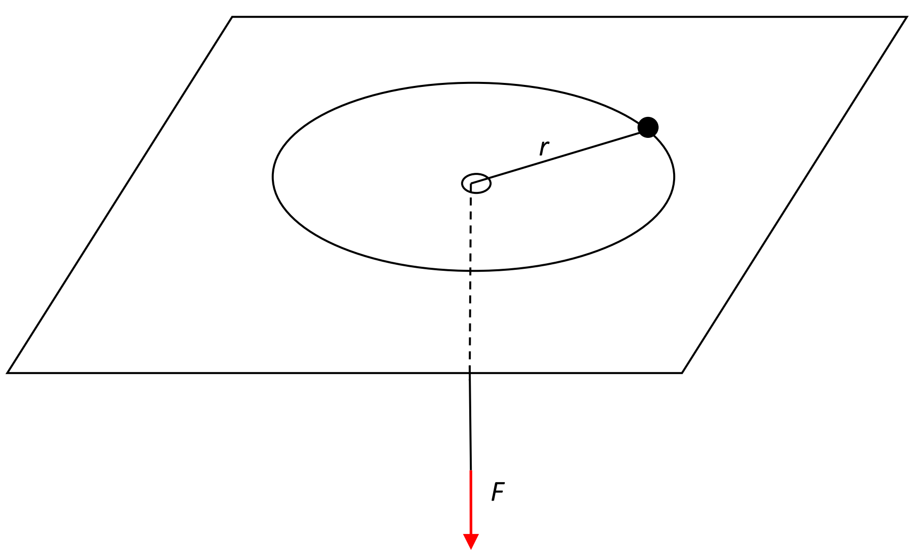
(e) The moment of inertia of a sphere of mass *M* and radius *R* about an axis through its center is $\frac{2}{5}MR^2$. The sphere is placed at the top of an inclined plane of height h and released from rest. The sphere rolls without slipping.



(i) Describe what is meant by "the sphere rolls without slipping".

(ii) Determine the linear speed of the ball when it reaches horizontal ground.

(f) A mass *m* rotates on a horizontal frictionless table in a circular path of radius *r*. The mass is attached to a string that goes through a hole in the table. A vertical force *F* acts on the string so that the radius of the path is constant. The kinetic energy of the ball is *K*.



(i) Show that $F = \frac{L^2}{mr^3}$ where *L* is the angular momentum of the mass.

(ii) The force *F* is increased slowly pulling the string down so that the radius of the circular path is halved. Determine the new kinetic energy of the mass.

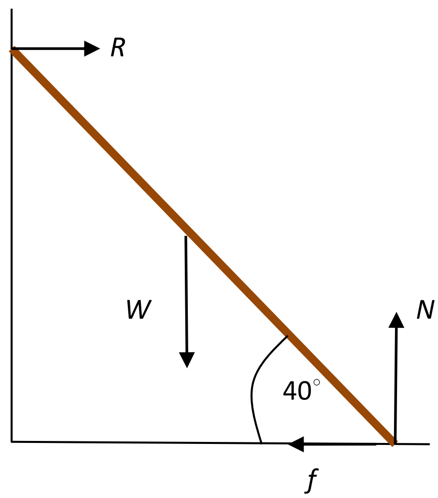
(iii) Determine the work done by *F* in terms of *K*.

Answers

(a) Split the ring into bits each of mass δm. Each bit is at the same distance *R* from the axis so $I=\sum(\delta m)R^2=R^2\sum(\delta m)=MR^2$.

(b)

(i)



(ii) Equilibrium of forces gives: $f=R$ and $N=W=250$ N. Taking torques about where the ladder touches the floor: $W\frac{L}{2}\cos 40^\circ=RL\sin 40^\circ$. Hence $R=\frac{W}{2}\cot 40^\circ=148.97\approx 150$ N. Hence $f\approx 150$ N.

(iii) The weight of the ladder effectively increases to 300 N. Then by (ii) $f=R=\frac{300}{2}\cot 40^\circ=179\approx 180$ N. The maximum frictional force that can develop is $f_{max}=0.70\times 300=210$ N so equilibrium is maintained.

(An additional point: you can easily show that any extra weight *w* attached to the middle will keep the ladder in equilibrium.)

(iv) Now $f=0.70\times 250=175$ N. Hence $R=175$ N. Taking torques about where the ladder touches the floor: $W\frac{L}{2}\cos\theta=RL\sin\theta$. Hence $\tan\theta=\frac{W}{2R}=\frac{250}{2\times 175}=0.714\Rightarrow\theta=35.54^\circ\approx 36^\circ$.

(c)

(i) It will not move because the resultant force is zero so the acceleration is zero.

(ii) $\tau=I\alpha\Rightarrow 120\times 0.80\times 2=35\times 0.80^2\times\alpha$, hence $\alpha=8.57\approx 8.6\text{ rad s}^{-2}$.

(iii) $\Delta L = \tau\Delta t = (120\times 0.80\times 2)\times 3.0 = 576 \approx 580 \text{ kg m}^2 \text{ s}^{-1}$.

(d) Angular momentum is a vector, but our guide does not give details of how to find the direction. Clearly, since the direction of rotation changes from counterclockwise to clockwise (as we look from above), the angular momentum vector must be pointing in the opposite direction to its original direction. The magnitude of the change of the angular momentum is then 15 – (–15) = 30 kg m^2 s^{-1}. Hence

$$\tau = \frac{\Delta L}{\Delta t} = \frac{30}{2.0} = 15 \text{ N m}.$$

(e)

(i) In one revolution the center of mass moves a distance equal to $2\pi R$.

(ii) $MgH = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 \Rightarrow MgH = \frac{1}{2}Mv^2 + \frac{1}{2}\frac{2MR^2}{5}(\frac{v}{R})^2 \Rightarrow gH = \frac{1}{2}v^2 + \frac{1}{5}v^2 = \frac{7}{10}v^2$. Hence

$$v^2 = \frac{10gH}{7}.$$

(f)

(i) $F = \frac{mv^2}{r} = \frac{m^2v^2r^2}{mr^3} = \frac{L^2}{mr^3}$.

(ii) Angular momentum is conserved because the force *F* has no torque about the axis through the vertical string. Hence, $mvr = mv'\frac{r}{2} \Rightarrow v' = 2v$. Hence $K' = 4K$

(iii) The increase in kinetic energy of the mass is $4K - K = 3K$. This is equal to the work done by the resultant force on the mass which is the force *F*.