Teacher notes
Topic A

Conservative forces

We often use the result that $W_{\text{ext}} = \Delta E_{\text{T}}$ where W_{ext} is the total work done by external forces and ΔE_{T} the change in the total mechanical energy. But what is an external force in this context?

A better statement would be that W_{ext} is the total work done by the non-conservative forces in the problem. So, what is a conservative and what is a non-conservative force?

Conservative forces are forces which are derived from a potential energy function *U* through $F = -\frac{dU}{dr}$ (I am being sloppy by ignoring proper vector notation here, but we don't want to get into vector calculus). **The potential energy is then part of the total mechanical energy of the system**.

For example, weight is derived from the gravitational energy function $U = mgh$: $-\frac{d(mgh)}{dh}$. The minus sign indicates that as the height *h* increases (we move away from the surface) the weight force is in the opposite direction i.e. vertically down.

If we are to move far away from the surface so that *g* varies, then the potential energy becomes $U = -\frac{GMm}{r}$ and the gravitational force is $F_{\text{g}} = -\frac{dU}{dr} = -\frac{GMm}{r^2}$. (The gravitational force is opposite to the direction in which *r* is increasing, hence the minus sign.)

In electricity, $U = \frac{kQq}{r}$ and $F_{\text{e}} = -\frac{dU}{dr} = \frac{kQq}{r^2}$ (charges enter with the correct sign).

Similarly, the tension force in a spring is given in terms of the potential energy function $U = \frac{1}{2}kx^2$. The tension force is then $T = -\frac{d(\frac{1}{2}kx^2)}{dx} = -kx$. Again, the minus sign indicates that the tension is opposite to the extension *x*.

All the above are conservative forces because they are the (negative) derivative of a potential energy function. The total energy of the system includes a potential energy function for each such force. Conservative forces have interesting properties the most important of which is that the work done by a conservative force in moving a body from A to B does not depend on the path taken from A to B. **Only** conservative forces have this property.

It is then no surprise that non-conservative forces are forces for which no potential energy function exists. Friction, drag and air resistance forces are typical examples. Being non-conservative means that the work done **does depend** on the path followed.

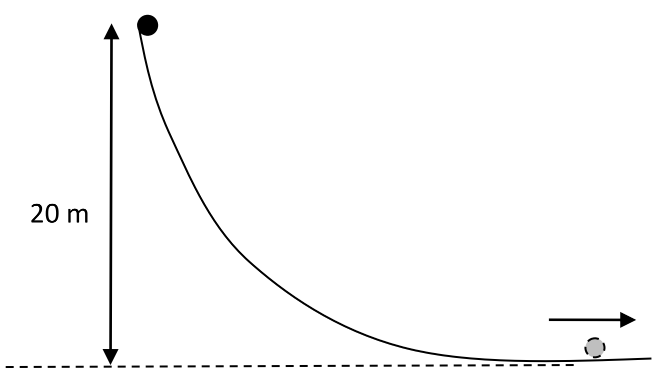
So, in using $W_{ext} = \Delta E_T$, what we mean by W_{ext} is simply the work done by *non-conservative* forces acting on the system.

If the problem involves conversion of internal energy into mechanical energy, then W_{ext} also includes this work even though this is not work from external forces. For example, an explosion converts chemical energy (a form of internal energy) into mechanical energy.

So, a better statement of $W_{ext} = \Delta E_T$ would be $W_{NC} = \Delta E_T$ where W_{NC} is the total work done by non-conservative forces, external or internal and ΔE_T is the change in total mechanical energy.

The typical example of an application of $W_{NC} = \Delta E_T$ is this:

A body of mass 4.0 kg slides from rest, down a curved incline of total length 35 m. The body changes its vertical height by 20 m.



At the bottom of the incline the speed of the body is 15 m s^{-1}. What is the frictional force acting on the body?

At the top the total energy is $E_T = mgh = 4.0 \times 10 \times 20 = 800$ J.

At the bottom the total energy is $E_T = \frac{1}{2}mv^2 = \frac{1}{2} \times 4.0 \times 15^2 = 450$ J.

The change in the total energy is $\Delta E_T = 450 - 800 = -350$ J.

This is the work done by the external forces. Weight is a conservative force (since there is a potential energy term in the total energy). Friction is non-conservative. (The normal force from the incline is normal to the displacement so it does zero work.) Hence, from $W_{\text{NC}} = \Delta E_{\text{T}}$:

$f \times 35 \times \cos 180^{\circ} = -350$ and so $f = 10$ N.

(But you must take care if you want to solve this problem using the work-kinetic energy relation! This approach says $W_{\text{net}} = \Delta E_{\text{K}}$, where W_{net} is the total work of **all** forces and ΔE_{K} the change in kinetic energy. Then,

$$\Delta E_{\text{K}} = \frac{1}{2}mv^2 - 0 = 450 \text{ J}$$

and

$$\begin{aligned} W_{\text{net}} &= W_N + W_{mg} + W_f \\ &= 0 + mgh - fs \\ &= 800 - f \times 35 \end{aligned}$$

Hence,

$450 = 800 - f \times 35$ leading to $f = 10$ N.)

Another application of $W_{\text{NC}} = \Delta E_{\text{T}}$ involving the conversion of internal energy into mechanical energy is the following:

A body of mass 10 kg at rest explodes into two pieces of 6.0 kg and 4.0 kg. The 4.0 kg moves with speed 6.0 m s^{-1}. How much internal (chemical) energy was converted into mechanical energy?

The initial mechanical energy was zero. By momentum conservation the 6.0 kg body moves with a momentum that is opposite to that of the 4.0 kg body i.e., a momentum of magnitude 24 N s. The total mechanical energy after the explosion is then (using $E_{\text{K}} = \frac{p^2}{2m}$)

$$\frac{24^2}{2 \times 4.0} + \frac{24^2}{2 \times 6.0} = 120 \text{ J}.$$

From $W_{\text{NC}} = \Delta E_{\text{T}}$, $W_{\text{NC}} = 120$ J, which is the chemical energy of the explosion that got transferred to mechanical energy.

Consider now the very similar problem of two masses of 4.0 kg and 6.0 kg that compress a spring between them. When they are let go the 4.0 kg mass moves away at 6.0 m s^{-1}. What was the elastic energy stored in the spring?

Just as before, the final mechanical energy of the system is 120 J. From $W_{NC} = \Delta E_T$ we find $0 = \Delta E_T = 120 - E_e$. This is because there are no non-conservative forces acting. Then $E_e = 120\,\text{J}$.

An extension

To a conservative force *F* corresponds a potential energy function *U* such that $F = -\frac{dU}{dr}$.

Similarly, to a conservative force *F* corresponds a field. For the gravitational force the field is *g* and for the electric force it is the electric field *E* such that $F_g = mg$ and $F_e = qE$.

And, to a potential energy function *U* corresponds a potential function *V* such that $U_g = mV_g$ and $U_e = qV_e$.

Then the relation $F = -\frac{dU}{dr}$ is equivalent to $g = -\frac{dV_g}{dr}$ and $g = -\frac{dV_e}{dr}$, the field is the negative derivative of the potential.